Starter Problems:

1. Simplify the following expressions completely.

(a) $$\dfrac{(-8p^5 r^{-2} w^{-4})^3}{(6p^{-1} r w^7)(4p^{-3} r^6 w^{-2})}$$

$$\dfrac{-8^3 p^{15} r^{-6} w^{-12}}{24\, p^{-4}\, r^7\, w^5}$$

$$\boxed{\dfrac{-64\, p^{19}}{3\, r^{13} w^{17}}}$$

(b) $(5x^5 y^{-2})^3 (2x^{-6} y^{10})$

$$125 x^{15} y^{-6} \cdot 2 x^{-6} y^{10}$$

$$\boxed{250 x^9 y^4}$$

Review:

If taking the root of the number you are given is not a __whole__ number, then we can use __perfect__ squares, cubes, etc. to break it down.

Background: List of perfect squares

1, 4, 9, __16__, __25__, __36__, __49__, __64__, __81__, __100__, etc.

Review Examples: Simplify each radical expression fully.

1.) $\sqrt{12}$

4   3

$\boxed{2\sqrt{3}}$

2.) $\sqrt{45}$

9   5

$\boxed{3\sqrt{5}}$

3.) $\sqrt{32}$

4   8

     4 2

$\boxed{4\sqrt{2}}$

4.) $\sqrt{75}$

25   3

$\boxed{5\sqrt{3}}$

5.) $\sqrt{60}$

6   10

3   2   5   2

$\boxed{2\sqrt{15}}$

6.) $\sqrt{128}$

2^7

$2^{7/2}$

$2^3 \sqrt{2}$

$\boxed{8\sqrt{2}}$

1

List of perfect cubes…

1, 8, ___27___, ___64___, ___125___, ___216___, etc.

Additional Examples: Simplify each radical expression fully.

10.) $\sqrt[3]{72}$

9   8
3^2 2^3

$2\sqrt[3]{9}$

11.) $\sqrt[3]{162}$

2   9^2
      3^4

$3\sqrt[3]{3 \cdot 2}$ $3\sqrt[3]{6}$

12.) $\sqrt[3]{-192}$

16   12
2^4 3 4
$2^6 3^1$ $-2^2\sqrt[3]{3}$

$-4\sqrt[3]{3}$

13.) $\sqrt[3]{250}$

125   2
5^3

$5\sqrt[3]{2}$

14.) $\sqrt[3]{-40}$

10   4
2 5 2^2
$2^3 \cdot 5$

$-2\sqrt[3]{5}$

15.) $\sqrt[3]{-432}$

144   3
12^2
$3^2 \cdot 2^4$ $3^3 \cdot 2^4$
$-3 \cdot 2\sqrt[3]{2}$

$-6\sqrt[3]{2}$

16.) $\sqrt[3]{120}$

12   10
3 4   5 2
$2\sqrt[3]{15}$

17.) $\sqrt[3]{189}$

63   3
7 9   3
      3
$3^3 \cdot 7$ $3\sqrt[3]{7}$

18.) $\sqrt[3]{-500}$

100   5
10   10
5 2   5 2
$2^2 \cdot 5^3$

$-5\sqrt[3]{4}$

19.) $\sqrt[4]{405}$

5
81 - 3^4

$3\sqrt[4]{5}$

20.) $\sqrt[5]{224}$

4   56
2^3 7
$2^5 \cdot 7$

$2\sqrt[5]{7}$

2

Note:

If the exponent of the variable is NOT divisible by __n__ , then we need to __break__ __up__ the variable and take out __as many__ as possible.

Examples: Simplify fully.

1.) $\sqrt{a^{10}b^9}$

$a^{10/2}\ b^{9/2}$

$\boxed{a^5 b^4 \sqrt{b}}$

2.) $\sqrt[3]{c^5d^{13}}$

$c^{5/3}\ d^{13/3}$

$\boxed{c\ d^4\ \sqrt[3]{c^2 d}}$

3.) $\sqrt[4]{h^{15}k^{28}}$

$h^{15/4}\ k^{28/4}$

$\boxed{h^3 k^7 \sqrt[4]{h^3}}$

4.) $\sqrt[6]{m^{14}n^3}$

$m^{14/6}\ n^{3/6}$

$\boxed{m^2 \sqrt[6]{m^2 n^3}}$

3

Finding the nth root of a number depends on whether the number is ___+___ or ___−___ and whether the root is __even__ or __odd__ :

a	n	$\sqrt[n]{a}$
+	even	$\cancel{\pm}^{+}$ ans
+	odd	1^{+} ans
−	even	can't do
−	odd	1^{-} ans.

We can also simplify radicals that contain both __numbers__ and __variables__!

To find the nth root of a variable, we need to __divide__ the __exponents__ of the variable by n...

5.) $\sqrt{300p^{27}}$

$\underset{100 \quad 3}{\wedge}$

$\boxed{10p^{13}\sqrt{3p}}$

6.) $\sqrt[3]{686r^{20}}$

$7^3 \cdot 2$

$\boxed{7r^6 \sqrt[3]{2r^2}}$

7.) $\sqrt[4]{2500x^{35}y^{13}}$

$5^4 \, 2^2 x^{35} y^{13}$

$\boxed{5x^8 y^3 \sqrt[4]{4x^3 y}}$

8.) $\sqrt[5]{1701u^{34}w^{11}}$

$3^5 \cdot 7 u^{34} w^{11}$

$3u^6 w^2 \sqrt[5]{7u^4 w}$

$\boxed{3u^6 w^2 \sqrt[5]{7u^4 w}}$

4

Examples: Simplify each expression fully.

1.) $\sqrt{81w^8}$

$3^4 w^6$

$\boxed{9w^4}$

2.) $\sqrt{49p^6}$

$7^2 p^6$

$\boxed{7p^3}$

3.) $\sqrt{196h^{12}k^{22}}$

$14^2 h^{12} k^{22}$

$\boxed{14h^6k^{11}}$

4.) $\sqrt[3]{216j^{15}}$

$6^3 j^{15}$

$\boxed{6j^5}$

5.) $\sqrt[3]{-729u^6}$

$\boxed{-9u^2}$

6.) $\sqrt[3]{64a^{21}b^{30}}$

$4^3 a^{21} b^{30}$

$\boxed{4a^7b^{10}}$

7.) $\sqrt[4]{625c^8}$

$5^4 c^8$

$\boxed{5c^2}$

8.) $\sqrt[4]{16d^{20}}$

$2^4 d^{20}$

$\boxed{2d^5}$

9.) $\sqrt[4]{2401x^{36}y^{16}}$

$7^4 x^{36} y^{16}$

$\boxed{7x^9y^4}$

10.) $\sqrt[5]{243a^{10}b^{35}}$

$3^5 a^{10} b^{35}$

$\boxed{3a^2b^7}$

11.) $\sqrt[6]{64c^{12}d^{30}}$

$2^6 c^{12} d^{30}$

$\boxed{2c^2d^5}$

12.) $\sqrt[7]{-78125h^{42}k^{21}}$

$-5^7 h^{42} k^{21}$

$\boxed{-5h^6k^3}$

5

Extra Practice: Directions: Simplify each expression.

1.) $\sqrt{256a^{20}}$

$\boxed{16a^{10}}$

2.) $\sqrt[3]{729b^{18}}$

$9^3 b^{18}$

$\boxed{9b^6}$

3.) $\sqrt[4]{1296c^{12}}$

$6^4 c^{12}$

$\boxed{6c^3}$

4.) $\sqrt[7]{-128d^{56}}$

$\cancel{-20} \quad -2^7 d^{56}$

$\boxed{-2d^8}$

5.) $\sqrt{25g^{16}h^{26}}$

$\boxed{5g^8h^{13}}$

6.) $\sqrt[3]{125j^{21}k^3}$

$\boxed{5j^7k}$

7.) $\sqrt[4]{81m^{20}n^{28}}$

$\boxed{3m^5n^7}$

8.) $\sqrt[5]{16807p^{10}r^{40}}$

$\boxed{7p^2r^8}$

9.) $\sqrt[6]{4096u^{48}v^{60}}$

$\boxed{4u^8v^{10}}$

10.) $\sqrt[7]{2187x^{35}y^{14}}$

$\boxed{3x^5y^2}$

6

Homework: Simplifying Radicals

Directions: Simplify fully by simplifying each expression in radical form.

1.) $\sqrt{m^9 n^{15}}$

$m^{9/2} n^{15/2}$

Answer: $\boxed{m^4 n^7 \sqrt{mn}}$

2.) $\sqrt{567 w^{26}}$

$3^4 \cdot 7 w^{26}$

Answer: $\boxed{9 w^{13} \sqrt{7}}$

3.) $\sqrt[3]{a^{16} b^2}$

$a^{16/3} b^{2/3}$

Answer: $\boxed{a^5 \sqrt[3]{a b^2}}$

4.) $\sqrt[3]{324 h^5 k^{21}}$

$2^2 \cdot 3 x^{?} y^{?}$

$3^{4/3} 2^{2/3} h^{5/3} k^{2/3}$

Answer: $\boxed{3 h k^7 \sqrt[3]{12 h^2}}$

$\boxed{4 x^5 y^6 \sqrt[4]{3 y^3}}$

5.) $\sqrt[4]{768 x^{20} y^{27}}$

$2^8 \cdot 3 x^{20} y^{27}$

Answer: $\boxed{4 x^5 y^6 \sqrt[4]{3 y^3}}$

6.) $\sqrt[5]{288 c^{18} d^5}$

$2^5 3^2 c^{18} d^5$

Answer: $\boxed{2 c^3 d \sqrt[5]{9 c^3}}$

7

7.) $\sqrt{192a^{17}b^{31}}$

$2^{6/2} \cdot 3^{1/2} a^{17/2} b^{31/2}$

Answer: $\boxed{8a^8 b^{15} \sqrt{3ab}}$

8.) $\sqrt[3]{135g^{11}}$

$3^3 5 g^{11}$

Answer: $\boxed{3g^3 \sqrt[3]{5g^2}}$

9.) $\sqrt[3]{864c^{16}d^{12}}$

$2^5 3^3 c^{16} d^{12}$

Answer: $\boxed{6c^5 d^4 \sqrt[3]{4c}}$

10.) $\sqrt[4]{1250h^{31}k^{24}}$

$2 \cdot 5^4 h^{31} k^{24}$

Answer: $\boxed{5h^7 k^6 \sqrt[4]{2h^3}}$

11.) $\sqrt[5]{2430m^{17}n^{44}}$

$2 \cdot 3^5 \cdot 5 m^{17} n^{44}$

Answer: $\boxed{3m^3 n^8 \sqrt[5]{10m^2 n^4}}$

12.) $\sqrt[6]{320p^{53}r^{10}}$

$2^6 \cdot 5 p^{53} r^{10}$

Answer: $\boxed{2p^8 r \sqrt[6]{5p^5 r^4}}$

8

Topic: **Rational Exponents** Date: _____

Starter Problems:

(1.) Simplify: $\sqrt[6]{729m^{24}n^{42}}$

$$3^6 m^{24} n^{42}$$

$$\boxed{3m^4n^7}$$

(2.) Simplify: $\sqrt{5x^6y^3z} \cdot \sqrt{12x^5y^{10}z^2}$

$$2^2 \cdot 3 \cdot 5 x^{11} y^{13} z^3$$

$$\boxed{2x^5y^6z\sqrt{15xyz}}$$

Review:

1.) $x^0 = 1$

2.) $x^1 = X$

3.) $x^{-1} = \dfrac{1}{X}$

4.) $x^{-3} = \dfrac{1}{X^3}$

5.) $8^{-2} = \dfrac{1}{64}$

6.) $3^{-5} = \dfrac{1}{243}$

Rule #1: $x^{1/a} = \sqrt[a]{X^1}$ $\dfrac{\text{power}}{\text{root}}$

Rule #2: $x^{b/a} = \sqrt[a]{X^b}$

Examples:

1.) $100^{1/2} = (10^2)^{1/2} = \boxed{10}$

2.) $625^{1/4} = (5^4)^{1/4} = \boxed{5}$

3.) $343^{1/3} = (7^3)^{\frac{1}{3}} = \boxed{7}$

4.) $(-1024)^{1/5} \ (-4^5)^{1/5} = \boxed{-4}$

5.) $64^{3/2} = (2^6)^{\frac{3}{2}} = 2^9 = \boxed{512}$

6.) $1296^{3/4} = (6^4)^{\frac{3}{4}} = \boxed{216}$

7.) $(-27)^{5/3} = (-3^{\cancel{3}})^{\frac{5}{\cancel{3}}} = \boxed{-243}$

8.) $(-32)^{6/5} = (-2^{\cancel{5}})^{\frac{6}{\cancel{5}}} = \boxed{64}$

9

Rule #3: $\quad x^{-1/a} = \dfrac{1}{x^{1/a}} = \dfrac{1}{\sqrt[a]{x}}$	Rule #4: $\quad x^{-b/a} = \dfrac{1}{x^{b/a}} \;\; \dfrac{1}{\sqrt[a]{x^b}}$

Examples:

9.) $2401^{-1/4} = (7^4)^{-\frac{1}{4}} = \boxed{\dfrac{1}{7}}$

10.) $729^{-1/3} = (9^3)^{-\frac{1}{3}} = \boxed{\dfrac{1}{9}}$

11.) $289^{-1/2} = (17^2)^{-\frac{1}{2}} = \boxed{\dfrac{1}{17}}$

12.) $(-7776)^{-1/5} = (-6^5)^{-\frac{1}{5}} = \boxed{\dfrac{-1}{6}}$

13.) $16^{-7/4} = (2^4)^{-\frac{7}{4}} = \boxed{\dfrac{1}{128}}$

14.) $4096^{-5/6} = (2^{12})^{-\frac{5}{6}} = 2^{-10} = \boxed{\dfrac{1}{1024}}$

15.) $(-512)^{-2/3} = (-8^3)^{-\frac{2}{3}} \;\; \boxed{\dfrac{1}{64}}$

16.) $(-243)^{-7/5} = (-3^5)^{-\frac{7}{5}} \;\; \boxed{\dfrac{1}{-2187}}$

Practice: Simplify each expression.

1.) $125^{\frac{1}{3}} \quad (5^3)^{1/3} = \boxed{5}$

2.) $36^{\frac{3}{2}} \quad (6^2)^{\frac{3}{2}} = \boxed{216}$

3.) $27^{\frac{4}{3}} \quad (3^3)^{4/3} = \boxed{81}$

4.) $4096^{\frac{3}{4}} \quad (8^4)^{\frac{3}{4}} = \boxed{512}$

5.) $512^{\frac{2}{3}} \quad (8^3)^{\frac{2}{3}} = \boxed{64}$

6.) $6561^{\frac{5}{8}} \quad (3^8)^{\frac{5}{8}} = \boxed{243}$

7.) $(-243)^{\frac{1}{8}} \quad (-3^5)^{1/5} = \boxed{-3}$

8.) $(-64)^{\frac{5}{3}} \quad (-2^6)^{\frac{5}{3}} = \boxed{-1024}$

$(-4)^5$

10

9.) $(-1024)^{\frac{2}{5}}$ $(-4)^{\frac{2}{5}}$ = $\boxed{16}$

10.) $(-216)^{\frac{4}{3}}$ $(-6^3)^{\frac{4}{3}}$ $\boxed{+1296}$

11.) $4096^{-\frac{1}{6}}$ $(8^4)^{-\frac{1}{6}}$ $(2^{12})^{-\frac{1}{6}}$ = $\boxed{\frac{1}{4}}$

12.) $81^{-\frac{5}{4}}$ $(9^2)^{-\frac{5}{4}}$ $(3^4)^{-\frac{5}{4}}$ = $\boxed{\frac{1}{243}}$

13.) $121^{-\frac{3}{2}}$ $(11^2)^{-\frac{3}{2}}$ = $\boxed{\frac{1}{1331}}$

14.) $625^{-\frac{7}{4}}$ $(5^4)^{-\frac{7}{4}}$ $\boxed{\frac{1}{78125}}$

15.) $64^{-\frac{5}{6}}$ $(8^2)^{-\frac{5}{6}}$ $(2^6)^{-\frac{5}{6}}$ $\boxed{\frac{1}{32}}$

16.) $(-1728)^{-\frac{1}{3}}$ $(-12^3)^{-\frac{1}{3}}$ $\boxed{\frac{-1}{12}}$

17.) $(-343)^{-\frac{4}{3}}$ $(-7^3)^{-\frac{4}{3}}$ $\boxed{\frac{1}{2401}}$

18.) $(-3125)^{-\frac{3}{5}}$ $(-5^5)^{-\frac{3}{5}}$ $\boxed{\frac{-1}{125}}$

19.) $(-8)^{-\frac{7}{3}}$ $(-2^3)^{-\frac{7}{3}}$ $\boxed{\frac{1}{-128}}$

20.) $(-2187)^{-\frac{2}{7}}$ $(-3^7)^{-\frac{2}{7}}$ $\boxed{\frac{1}{9}}$

11

$$(x^m)^n = \quad X^{m \cdot n} \qquad \frac{x^m}{x^n} = \quad X^{m-n} \qquad x^m \cdot x^n = \quad X^{m+n}$$

New Examples:

1.) $4^{\frac{1}{3}} \cdot 4^{\frac{2}{3}} = \boxed{4}$

2.) $5^{\frac{1}{4}} \cdot 5^{\frac{3}{2}} = \quad \frac{1}{4} + \frac{3}{2} \times 2$

$\frac{1}{4} + \frac{6}{4}$

$5^{\frac{1}{4} + \frac{6}{4}} \quad \boxed{5^{\frac{7}{4}}}$

3.) $7^{\frac{5}{6}} \cdot 7^{\frac{3}{4}} = 7^{\frac{10}{12} + \frac{9}{12}}$

$\boxed{7^{\frac{19}{12}}}$

4.) $\sqrt{3} \cdot \sqrt[4]{3} =$

$3^{\frac{1}{3}} \quad 3^{\frac{1}{4}} = \boxed{3^{\frac{7}{12}}}$

5.) $\sqrt[5]{6} \cdot \sqrt[3]{6} =$

$6^{\frac{1}{5} + \frac{1}{3}} \quad \boxed{6^{\frac{8}{15}}}$

Examples (cont.):

6.) $\frac{2^{\frac{7}{5}}}{2^{\frac{4}{5}}} = 2^{\frac{7}{5} - \frac{4}{5}}$

$\boxed{2^{\frac{3}{5}}}$

7.) $\frac{3^{\frac{1}{2}}}{3^{\frac{9}{4}}} = 3^{\frac{1}{2} - \frac{9}{4}} \quad 3^{\frac{-7}{4}} \quad \boxed{\frac{1}{3^{\frac{7}{4}}}}$

8.) $\frac{\sqrt[3]{10}}{\sqrt[4]{10}} = 10^{\frac{1}{3} - \frac{1}{4}}$

$\boxed{10^{\frac{1}{12}}}$

9.) $\frac{\sqrt[5]{8}}{\sqrt{2}} = 2^{\frac{3}{5} - \frac{1}{2}}$

$2^{\frac{6}{10} - \frac{5}{10}} \quad \boxed{2^{\frac{1}{10}}}$

10.) $\frac{\sqrt[5]{9}}{\sqrt[4]{27}} = \frac{3^{\frac{2}{5}}}{3^{\frac{3}{4}}} \quad 3^{\frac{2}{5} - \frac{3}{4}}$

$3^{\frac{8}{20} - \frac{15}{20}} \quad 3^{\frac{-7}{20}} \quad \boxed{\frac{1}{3^{\frac{7}{20}}}}$

12

Simplify each expression completely.

1.) $7776^{\frac{1}{5}}$
$\left(6^5\right)^{1/5} = \boxed{6}$

2.) $64^{-\frac{1}{2}}$
$\left(2^6\right)^{-\frac{1}{2}} \quad \dfrac{1}{2^3} = \boxed{\dfrac{1}{8}}$

3.) $81^{\frac{5}{4}}$
$\left(3^4\right)^{\frac{5}{4}} = \boxed{243}$

4.) $343^{-\frac{2}{3}}$
$\left(7^3\right)^{-\frac{2}{3}} \quad \dfrac{1}{(7)^2} = \boxed{\dfrac{1}{49}}$

5.) $(-1024)^{\frac{3}{5}}$
$\left(-4^5\right)^{\frac{3}{5}} = \boxed{-64}$

6.) $(-125)^{-\frac{4}{3}}$
$\left(-5^3\right)^{-\frac{4}{3}} \quad \dfrac{1}{(-5)^4} = \boxed{\dfrac{1}{625}}$

7.) $4^{\frac{2}{3}} \cdot 4^{\frac{5}{2}}$
$4^{\frac{2}{3} + \frac{5}{2}}$
$4^{\frac{4}{6} + \frac{15}{6}} \qquad \boxed{4^{19/6}}$

8.) $\dfrac{7^{\frac{3}{4}}}{7^{\frac{1}{5}}}$
$7^{\frac{3}{4} - \frac{1}{5}}$ (×5 and ×4)
$7^{\frac{15}{20} - \frac{4}{20}} \qquad \boxed{7^{11/20}}$

9.) $\sqrt[4]{5} \cdot \sqrt[3]{5}$
$5^{1/4 + 1/3} = \boxed{5^{7/12}}$

10.) $\dfrac{\sqrt[6]{3}}{\sqrt{3}}$
$\dfrac{3^{1/6}}{3^{1/2}}$
$3^{1/6 - 1/2 \times 3} \qquad 3^{-2/6} \qquad \boxed{\dfrac{1}{3^{1/3}}}$

11.) $\sqrt[3]{25} \cdot \sqrt[4]{125}$
$5^{2/3} \cdot 5^{3/4}$
$5^{\frac{8}{12} + \frac{9}{12}} = \boxed{5^{17/12}}$

12.) $\dfrac{\sqrt{343}}{\sqrt[5]{49}}$
$\dfrac{\left(7^3\right)^{1/2}}{\left(7^2\right)^{1/5}}$
$7^{3/2 - 2/5}$ (×5 and ×2)
$7^{\frac{15}{10} - \frac{4}{10}} \qquad \boxed{7^{11/10}}$

Directions: Simplify each expression

1.) $(7a^3b^{-5})(4a^2b)$

$28a^5b^{-4}$

$$\boxed{\frac{28a^5}{b^4}}$$

2.) $\dfrac{(10m^5n^{-1})(6m^{-2}n^{-4})}{4mn^{-9}}$

$$\frac{60m^3n^{-5}}{4mn^{-9}}$$

$$\boxed{15m^2n^4}$$

3.) $(5c^4d^{-2})^{-3}$

$5^{-3}c^{-12}d^6$

$$\boxed{\frac{d^6}{125c^{12}}}$$

4.) $\sqrt[3]{243} \cdot \sqrt{27}$

$(3^5)^{1/3}(3^3)^{1/2}$

$3^{\frac{5}{3} \times \frac{2}{2}} + 3^{\frac{3}{2} \times 3}$

$3^{\frac{10}{6}+\frac{9}{6}}$

$$\boxed{3^{19/6}}$$

5.) $\dfrac{\sqrt[6]{32}}{8^{\frac{4}{5}}}$ $\dfrac{2^{5/6}}{(2^3)^{\frac{4}{5}}}$

$2^{\frac{5}{6} \times 5 - \frac{12}{5} \times 6}$

$2^{\frac{25}{30} - \frac{72}{30}}$

$$\boxed{\frac{1}{2^{47/30}}}$$

6.) $625^{-\frac{1}{4}}$

$(5^4)^{-\frac{1}{4}}$

$$\boxed{\frac{1}{5}}$$

7.) $(-7776)^{\frac{3}{5}}$

$(-6^5)^{\frac{3}{5}}$ $(-6)^3$

$$\boxed{-216}$$

8.) $(-64)^{-\frac{4}{3}}$

$(-4^3)^{-\frac{4}{3}}$ $\dfrac{1}{(-4)^4} = \boxed{\dfrac{1}{256}}$

Write the expression in radical form.

A. $x^{\frac{2}{3}}$

$$\boxed{\sqrt[3]{x^2}}$$

B. $-2^{\frac{3}{4}}$

$-\sqrt[4]{2^3}$

$$\boxed{-\sqrt[4]{8}}$$

C. $(-2)^{\frac{2}{3}}$

$\sqrt[3]{(-2)^2}$

D. $7(a+1)^{-\frac{1}{2}}$

$$\boxed{\frac{7}{\sqrt{a+1}}}$$