Notes Topic: 9.3/4 Solving Quadratic Equations Date: ______

Starter(s):

1.) Find the vertex for the quadratic function.

$y = -x^2 + 8x + 9$

$x = \frac{-8}{2(-1)} = 4$

$-(4)^2 + 8(4) + 9$

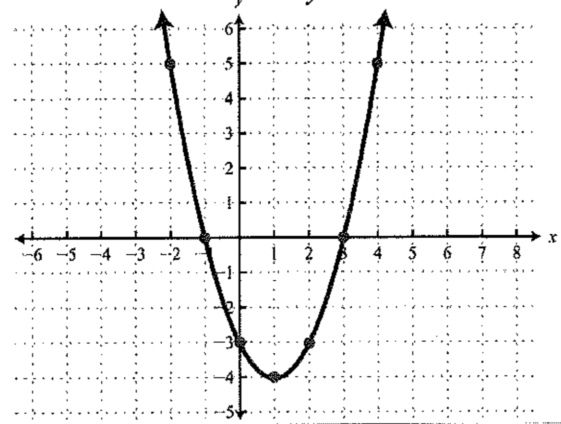
$-16 + 32 + 9$

$16 + 9 = 25$

Answer: (4, 25)

2. Answer the following questions below.

$y = x^2 - 2x - 3$



Use the graph above to answer the questions

(a) Vertex: (1, -4)

(b) Axis of symmetry: x = 1

(c) Domain: ℝ

(d) Range: $y \geq -4$

(e) Y-Intercept: (0, -3)

(f) X-Intercepts: (-1, 0) and (3, 0)

Background:

A parabola one y-intercept, which you can find by looking at the value of c.

Examples:

What is the y-intercept to the following quadratic functions?

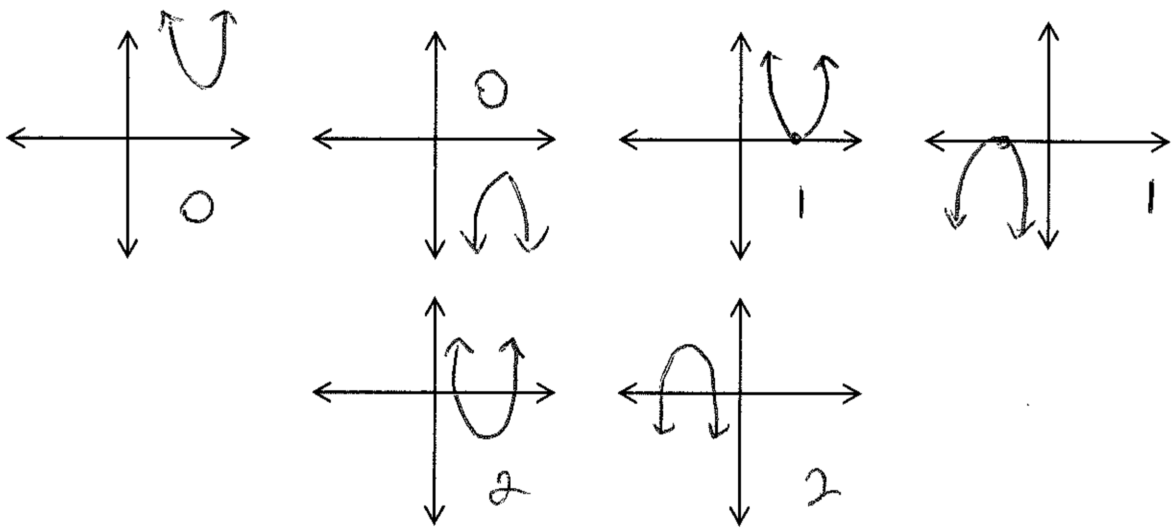
1.) $y = x^2 - x$ Answer: 0

2.) $y = x^2 + 4x - 16$ Answer: -16

3.) $y = x^2$ Answer: 0

Notes:

A parabola can have 0, 1, or 2 x-intercepts depending on whether the Vertex is above or below or on the X-axis and whether the parabola opens up or down.



Big Idea:

We can determine how many Solutions (zeros) a quadratic equation has based on how many X-intercepts the graph of its corresponding function would have.

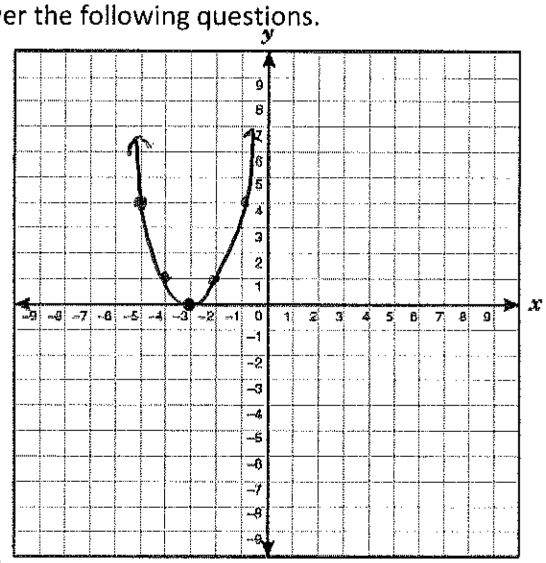
For #1 – 2, graph each quadratic function *algebraically* and answer the following questions.

1.) $y = x^2 + 6x + 9$

$$x = \frac{-6}{2(1)} = -3$$

$$(-3)^2 + 6(-3) + 9$$

$$9 - 18 + 9 = 0$$



Domain: $\mathbb{R}$

Range: $y \geq 0$

Vertex: $(-3, 0)$

Axis of symmetry: $x = -3$

Y-intercept: $(0, 9)$

# of X-Intercepts: 1, $(-3, 0)$

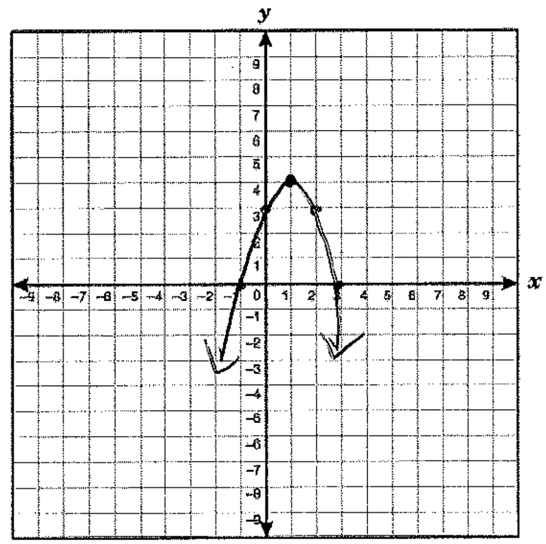
Min OR Max: min

2.) $y = -x^2 + 2x + 3$

$X = \frac{-2}{2(-1)} = 1$

$-(1)^2 + 2(1) + 3$

$-1 + 2 + 3 = 4$



Domain: $\mathbb{R}$

Range: $y \le 4$

Vertex: (1,4)

Axis of symmetry: $x = 1$

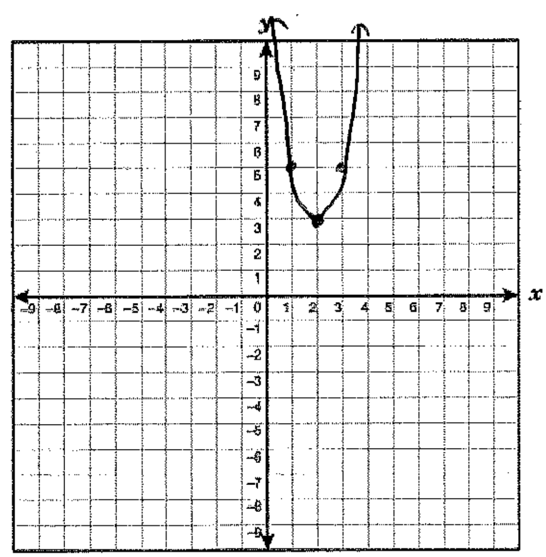
Y-intercept: (0,3)

# of X-Intercepts: 2, (-1,0) and (3,0)

Min OR Max: max

YOU TRY:

3.) $y = 2(x-2)^2 + 3$



Domain: $\mathbb{R}$

Range: $y \ge 3$

Vertex: (2,3)

Axis of symmetry: $x = 2$

Y-intercept: (0, 11)

# of X-Intercepts: none

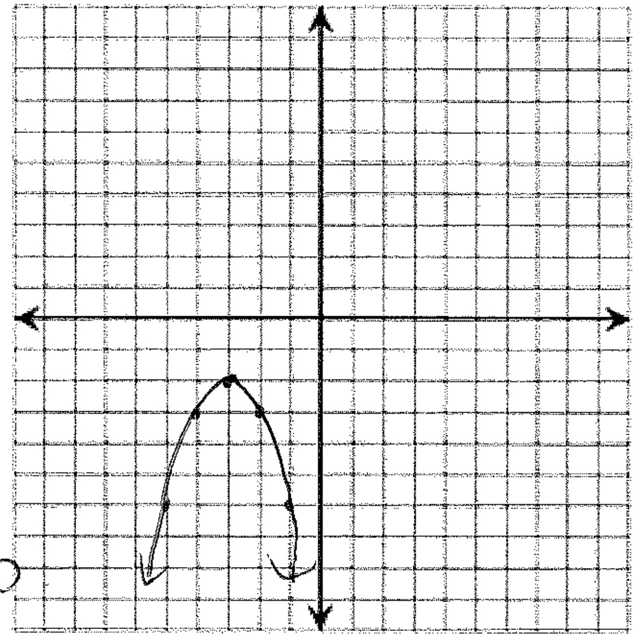
Min OR Max: min

Name: ____________________ Date: __________ Block: ______

Quadratics #4 **EXTRA PRACTICE**

Graph the following quadratic functions. Remember to look for five points!

1. $y = -x^2 + -6x - 11$



$x = \frac{6}{2(-1)} = -3$

$-(-3)^2 - 6(-3) - 11$

$-9 + 18 - 11 = -2$

Domain: ℝ

Range: $y \le -2$

Vertex: $(-3, -2)$

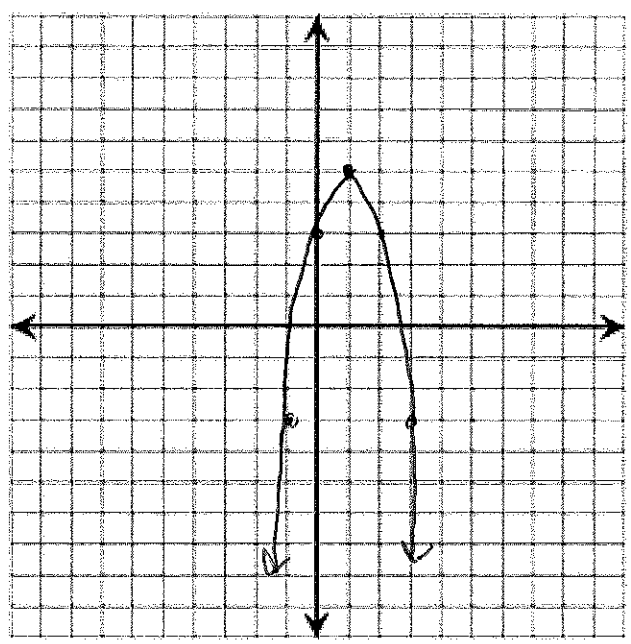
Axis of symmetry: $x = -3$

Y-intercept: $(0, -11)$

# of X-Intercepts: none, 0

Min OR Max: max

2. $y = -2x^2 + 4x + 3$



Domain: ℝ

Range: $y \le 5$

Vertex: $(1, 5)$

Axis of symmetry: $x = 1$

Y-intercept: $(0, 3)$

# of X-Intercepts: 2, between $x = -1$ and $x = 0$

$x = 2$ and $x = 3$

Min OR Max: max

3. For $y = 4x^2 + 16x + 19$

a) Find the vertex and determine if it represents a maximum or minimum point.

$$x = \frac{-16}{2(4)} = -2$$

$$4(-2)^2 + 16(-2) + 19$$
$$16 - 32 + 19$$
$$-16 + 19 = 3$$

$\underline{\text{min}}$

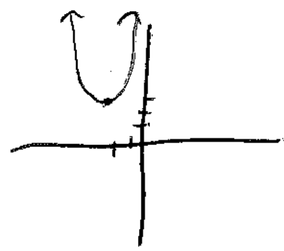
$\boxed{(-2, 3)}$

b) Find the domain and range.

$\mathbb{R}$ $\quad y \geq 3$

c) Determine how many x-intercepts the graph would have.

0



4. For $y = \frac{1}{2}x^2 - 6x + 15$

a) Find the vertex and determine if it represents a maximum or minimum point.

$$x = \frac{6}{2(\frac{1}{2})} = 6$$

$\underline{(6, -3)}$

$\underline{\text{min}}$

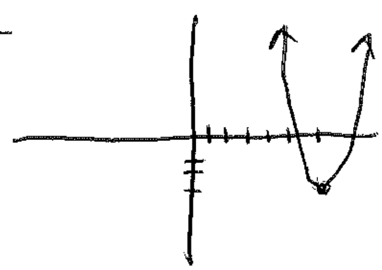
$$y = \frac{1}{2}(6)^2 - 6(6) + 15$$
$$18 - 36 + 15 = -3$$

b) Find the domain and range.

$\mathbb{R}$ $\quad y \geq -3$

c) Determine how many x-intercepts the graph would have.

2



POTD:

A golf ball hit on the moon with an initial speed of 18 feet per second has a height, h, in feet given by the function $h(t) = -3t^2 + 18t$, where t represents the number of seconds after the ball was hit.

(a) If the ball reaches its greatest height after 3 seconds, what is this greatest height?

$h(3) = -3(3)^2 + 18(3)$

$-27 + 54 = 27 \text{ ft}$

(b) At what point(s) in time is the ball at a height of 24 feet?

At 2 and 4 seconds

$-3t^2 + 18t - 24 = 0$

$-3(t^2 - 6t + 8) = 0$

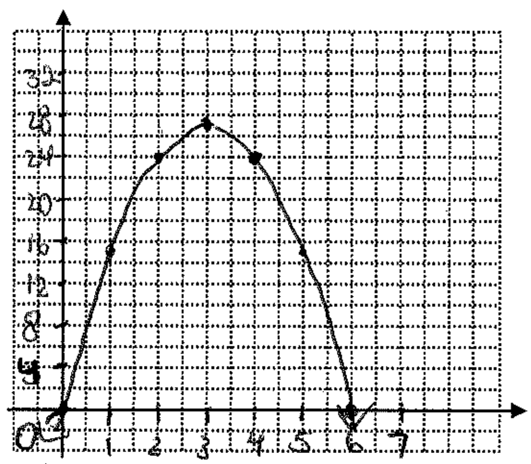
$-3(t-4)(t-2) = 0$

$t = 2, 4$

(c) Create a graph of the function on the grid below for all values of t on the interval $0 \le t \le 6$.

Make sure to properly label your axes.

t	$h(t)$
0	0
1	15
2	24
3	27
4	24
5	15
6	0



(d) How many seconds does it take for the ball to hit the ground?

6 seconds

(e) Looking at your answer(s) to part (d), is there another way to find when the ball will hit the ground *algebraically*? (Hint: What is the height of the ball when it is on the ground and which variable represents height in the equation?) Explain how you get your answer.

$-3t^2 + 18t = 0$

$-3t(t-6) = 0$

$t = 0, \; t = 6$

Big Idea:

If a quadratic equation has 1 or 2 solutions, then sometimes we can find those solutions by factoring.

Steps: To find the solutions of a quadratic equation by factoring...

#1 – Move all the terms to the left side of the equation using addition or subtraction so the right side of the equation is now 0.

#2 – If possible, divide each term by the a value.

#3 – factor!

#4 – Set each parenthesis equal to 0 and solve.

Examples:

1.) $a^2 - 14a + 48 = 0$

$(a-8)(a-6)=0$

$a-8=0 \quad a-6=0$

$a=\{6,8\}$

2.) $h^2 - 5h - 26 = 10$

$h^2-5h-36=0$

$(h+4)(h-9)=0$

$h=\{-4,9\}$

3.) $p^2 + 2p = 120$

$p^2+2p-120=0$

$(p+12)(p-10)=0$

$p=\{-12,10\}$

4.) $r^2 = 7r - 6$

$r^2-7r+6=0$

$(r-6)(r-1)=0$

$r=\{1,6\}$

5.) $-w^2 + 8w + 57 = 2w - 15$

$-2w + 15$

$-w^2 + 6w + 72 = 0$

$-1(w^2 - 6w - 72) = 0$

$-1(w+6)(w-12) = 0$

$w = \{-6, 12\}$

6.) $24m - 48 = 3m^2$

$-3m^2 + 24m - 48 = 0$

$-3(\sqrt{m^2} - 8m + \sqrt{16}) = 0$

$m \quad 4$

$2 \cdot m \cdot 4 = 8m$

$-3(m-4)^2 = 0$

$m = 4$

7.) $-2n^2 - 240 = 46n$

$-2n^2 - 46n - 240 = 0$

$-2(n^2 + 23n + 120) = 0$

$-2(n+15)(n+8) = 0$

$n = \{-15, -8\}$

8.) $c^2 = 36$

$c^2 - 36 = 0$

$(c+6)(c-6) = 0$

$c = \{-6, 6\}$

9.) $6k^2 - 17k + 5 = 0$

$6k^2 - 15k - 2k + 5 = 0$

$3k(2k-5) - 1(2k-5) = 0$

$(2k-5)(3k-1) = 0$

$2k - 5 = 0 \quad k = \frac{5}{2}$ | $3k - 1 = 0 \quad k = \frac{1}{3}$

10.) $12d^2 = 11d + 15$

$12d^2 - 11d - 15 = 0$

$12d^2 - 20d + 9d - 15 = 0$

$4d(3d-5) + 3(3d-5) = 0$

$(4d+3)(3d-5) = 0$

$d = \{-3/4, 5/3\}$

Result: $k = \{\frac{1}{3}, \frac{5}{2}\}$

The solutions you just ~~found for all the above~~ questions represent the x-intercepts or zeros. This is where the quadratic will cross the x-axis.

Name: ______________________ Date: ____________ Block:______

Quadratics: Solve by Factoring

Directions: Solve each quadratic equation for the given variable (show all your work).

1.) $a^2 + 19a + 84 = 0$

$(a+12)(a+7)=0$

$a=\{-12,-7\}$

2.) $h^2 - h - 50 = 22$

$h^2-h-72=0$

$(h-9)(h+8)=0$

$h=\{-8,9\}$

3.) $p^2 + 6p = 135$

$p^2+6p-135=0$

$(p+15)(p-9)=0$

$p=\{-15,9\}$

4.) $r^2 = 20r - 100$

$\sqrt{r^2}-20r+\sqrt{100}=0$

$r \quad\quad 10$

$2 \cdot r \cdot 10 = 20r$

$(r-10)^2=0$

$r=10$

5.) $-2w^2 - 24w + 200 = 72$

$-2w^2-24w+128=0$

$-2(w^2+12w-64)=0$

$-2(w+16)(w-4)=0$

$w=\{-16,4\}$

6.) $36m = 144 - 4m^2$

$4m^2+36m-144=0$

$4(m^2+9m-36)=0$

$4(m+12)(m-3)=0$

$m=\{-12,3\}$

7.) $180 = 2n^2 - 18n$

$-2n^2 + 18n + 180 = 0$

$-2(n^2 - 9n - 90) = 0$

$-2(n - 15)(n + 6) = 0$

$n = \{-6, 15\}$

8.) $-5c^2 + 62 = 17$

$-5c^2 + 45 = 0$

$-5(c^2 - 9) = 0$

$-5(c + 3)(c - 3) = 0$

$c = \{-3, 3\}$

9.) $10k^2 - 41k + 21 = 0$

$10k^2 - 35k - 6k + 21 = 0$

$5k(2k - 7) - 3(2k - 7) = 0$

$(5k - 3)(2k - 7) = 0$

$k = \{3/5, 7/2\}$

10.) $4d^2 + 15d = 54$

$4d^2 + 15d - 54 = 0$

$4d^2 + 24d - 9d - 54 = 0$

$4d(d + 6) - 9(d + 6) = 0$

$(4d - 9)(d + 6) = 0$

$d = \{-6, 9/4\}$

Name: Key
Solving Quadratic Equations

Solving Quadratic Equations by Factoring

Warm-up: Factor.

1. $3x^2 + 7x + 4$

$3x^2 + 3x + 4x + 4$

$3x(x+1) + 4(x+1)$

$(3x+4)(x+1)$

2. $2x^2 - 13x + 21$

$2x^2 - 6x - 7x + 21$

$2x(x-3) - 7(x-3)$

$(2x-7)(x-3)$

3. $x^2 + 4x - 45$

$(x+9)(x-5)$

4. $5x^2 + 34x - 7$

$5x^2 + 35x - 1x - 7$

$5x(x+7) - 1(x+7)$

$(5x-1)(x+7)$

To solve a quadratic equation:

1. Use the zero-product property by
2. Factor
3. Set each factor equal to zero

Examples Solve.

1. $16x^2 + 8x = 0$

$8x(2x+1) = 0$

$x = \{-8, -\frac{1}{2}\}$

2. $x^2 + 16x + 64 = 0$

$2 \cdot x \cdot 8 = 16x$

$(x+8)^2 = 0$

$x = -8$

3. $x^2 = 81$

$x^2 - 81 = 0$

$(x+9)(x-9) = 0$

$x = \{9, 9\}$

4. $4x^2 - 29 = 0$

$4x^2 = 29$

$x = \pm\sqrt{\frac{29}{4}}$

$x = \frac{\pm\sqrt{29}}{2}$

5. $x^2 + 9x = -20$

$x^2 + 9x + 20 = 0$

$(x+4)(x+5) = 0$

$x = \{-5, -4\}$

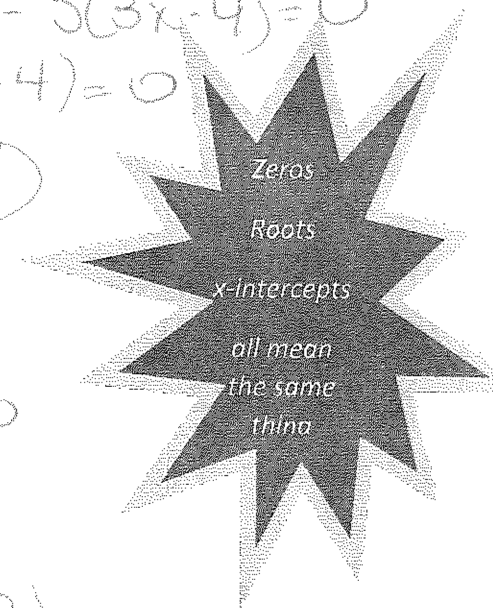
6. $6x^2 - 23x + 20 = 0$

$6x^2 - 8x - 15x + 20 = 0$

$2x(3x-4) - 5(3x-4) = 0$

$(2x-5)(3x-4) = 0$

$x = \{4/3, 5/2\}$



Write a quadratic equation with the given zeros:

7. $x = 2, -3$

$(x-2)(x+3) = 0$

$x^2 + x - 6 = 0$

8. $x = 0, -4$

$x(x+4) = 0$

$x^2 + 4x = 0$

9. $x = 3$

$(x-3)^2 = 0$

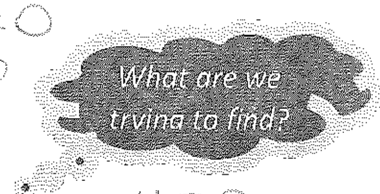
10. $x = \frac{3}{4}, \frac{2}{7}$

$\left(x - \frac{3}{4}\right)\left(x - \frac{2}{7}\right)$

$(4x-3)(7x-2) = 0$

$28x^2 - 29x + 6 = 0$

11. The height of a javelin in feet is modeled by $h(t) = -16t^2 + 79t + 5$, where t is the time in seconds after the javelin is thrown. How long is it in the air?



"$y = 0$ hits the ground"

$-16t^2 + 79t + 5 = 0$

$-16t^2 + 80t - t + 5 = 0$

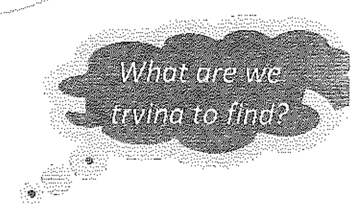
$-16t(t-5) - 1(t-5) = 0$

$(-16t - 1) = 0 \qquad t - 5 = 0$

$\cancel{t = -1/16} \qquad t = 5$

5 seconds

12. Juan recorded his brother bungee jumping from a height of 1100 feet. At the time the cord lifted his brother back up, he was 76 feet above the ground. If Juan started recording as soon as his brother fell, how much time elapsed when the cord snapped back? Use $f(t) = -16t^2 + c$, where c is the height in feet.



$76 = -16t^2 + 1100$

$-1024 = -16t^2$

$64 = t^2$

$\pm 8 = t$

$t = 8$ seconds

13. A company plans to build a large multiplex theater. The financial analyst told her manger that the profit function for their theater was $P(x) = -x^2 + 48x - 512$, where x is the number of movie ~~sereean~~ screens, and $P(x)$is the profit earned in thousands of dollars. Determine the range of production of movie screens that will guarantee that the company will not lose money.

What are we trying to find?

$+x^2 \mp 48x + 512 = 0$

$+x^2 - 16x - 32x + 512 = 0$

$+x(x \mp 16) - 32(x \mp 16) = 0$

$+x - 32 = 0 \quad x \mp 16 = 0$

$x = +32$

$x = +16$

between 16 and 32 movie screens.

Your turn...

a. $x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$x = \{-3, 7\}$

d. $4x^2 - 12x + 9 = 0$

$2 \cdot 2x \cdot 3 = 12x$

$(2x-3)^2 = 0$

$x = \frac{3}{2}$

g. $x^2 - 5x = 24$

$x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$

$x = \{-3, 8\}$

b. $15x^2 - 8x + 1 = 0$

$15x^2 - 5x - 3x + 1 = 0$

$5x(3x-1) - 1(3x-1) = 0$

$(5x-1)(3x-1) = 0$

$x = \{1/5, 1/3\}$

e. $81x^2 - 9x = 0$

$9x(9x-1) = 0$

$x = \{0, \frac{1}{9}\}$

h. $-12x^2 + 8x + 15 = 0$

$+12x^2 - 18x + 10x \mp 15 = 0$

$6x(2x-3) + 5(2x-3) = 0$

$(6x+5)(2x-3) = 0$

$x = \{-5/6, 3/2\}$

c. $20x^2 + 15x = 0$

$5x(4x+3) = 0$

$x = \{-3/4, 0\}$

f. $x^2 - 11x + 30 = 0$

$(x-5)(x-6) = 0$

$x = \{5, 6\}$

i. $6x^2 - 3x = 0$

$3x(x-1) = 0$

$x = \{0, 1\}$

j. After analyzing the market, a company that sells websites determines the profitability of their product was modeled by $P(x) = -16x^2 + 368x - 2035$, where x is the rpice of each website and $P(x)$ is the company's profit. Determine the price range of the websites that will be profitable for the company.

$16x^2 - 368x + 2035 = 0$

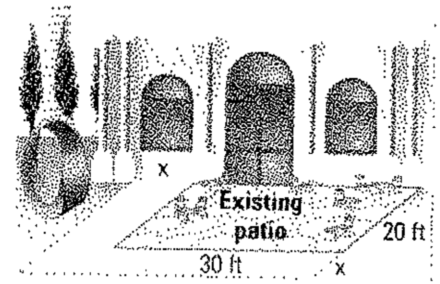
$16 \cdot 2035 = 32560$

$x = 9.25 \quad x = 13.75$

Price of website between \$9.25 and \$13.75 to stay w/in budget.

More Word Problems!

14. A museum has a café with a rectangular patio that measures 30 feet by 20 feet. The museum wants to add 464 square feet to the area of the patio by expanding the existing patio on two sides, as shown.



a. Find the area of the existing patio.

$30 \times 20 = 600 \text{ ft}^2$

b. Write an equation that you can use to find the value of *x*.

$(x+20)(x+30) = 600 + 464$

$x^2 + 50x - 464 = 0$

c. Solve the equation and interpret the answer.

$(x-8)(x+58) = 0$

$x = 8 \quad x = \cancel{-58}$

expand sides of patio by 8 ft

15. The product of two consecutive integers is 56. Find the integers.

$x(x+1) = 56$

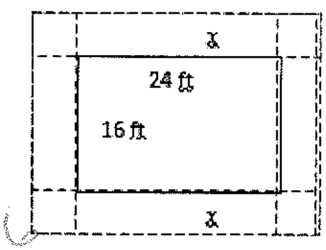
$x^2 + x - 56 = 0$

$(x+8)(x-7) = 0$

$\boxed{7, 8}$

YOU TRY:

k. Suppose you want to expand the garden shown at the right by planting a border of flowers. The border will be of the same width around the entire garden. The flowers you bought will fill an area of 276 ft^2. How wide should the border be?



$(2x+16)(2x+24) = 384 + 276$

$4x^2 + 80x + 384 = 384 + 276$

$4x^2 + 80x - 276 = 0$

$4(x^2 + 20x - 69) = 0$

$(x+23)(x-3) = 0$

$x = 3 \text{ ft}$

l. The product of two consecutive odd integers is 99. Find the integers.

$x(x+2) = 99$

$x^2 + 2x - 99 = 0$

$(x+11)(x-9) = 0$

9, 11

m. At last year's school fair, an 18 by 15 foot rectangular section of land was roped off for a dunking booth. The length and width of the section will each be increased by *x* feet for this year's fair in order to triple the original area. Write and solve an equation to find the value of *x*. What length of rope will be needed to enclose the new section?

$(x+18)(x+15) = 3(18)(15)$

$x^2 + 33x + 270 = 810$

$x^2 + 33x - 540 = 0$

$(x+45)(x-12) = 0$

12 ft