9.6 Solving Quadratic Equations by Using the Quadratic Formula – Key

Warm-up: Evaluate for x^2

$x = -1$	$x = 6$
$x^2 = (-1)^2 = 1$	$x^2 = (6)^2 = 36$
$x = -3$	$x = 4$
$x^2 = (-3)^2 = 9$	$x^2 = (4)^2 = 16$

Evaluate for $b^2 - 4ac$

$a = 1, b = 3, c = 4$	$a = 1, b = -3, c = 2$
$3^2 - 4(1)(4)$ $9 - 16 = -7$	$(-3)^2 - 4(1)(2)$ $9 - 8 = 1$
$a = 4, b = -3, c = 7$	$a = -1, b = -5, c = -7$
$(-3)^2 - 4(4)(7)$ $9 - 112 = -103$	$(-5)^2 - 4(-1)(-7)$ $25 - 28 = -3$

Standard Form of a Quadratic Equation:

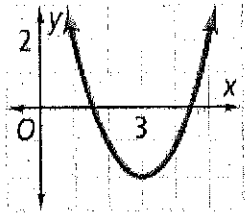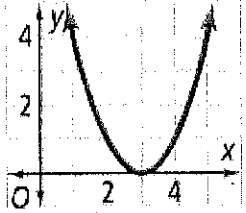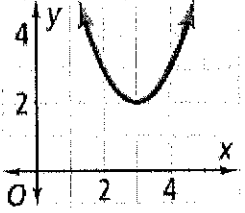$$y = ax^2 + bx + c$$

Discriminant

$$b^2 - 4ac$$

take note ➤ **Key Concept   Using the Discriminant**

Discriminant	greater than 0	equals 0	less than zero
Example	$x^2 - 6x + 7 = 0$	$x^2 - 6x + 9 = 0$	$x^2 - 6x + 11 = 0$
	$b^2 - 4ac = 8$	$b^2 - 4ac = 0$	$b^2 - 4ac = -8$
Number of Solutions	2 real Solutions	one real Solution	no real Solutions

How many real solutions does the quadratic equation have? $b^2 - 4ac$

1. $3x^2 + 10 + 2 = 0$

$a = 3$
$b = 10$ $10^2 - 4(3)(2)$
$c = 2$ 76

two real solutions

2. $9x^2 - 6x = -1$

$9x^2 - 6x + 1$
$a = 9$ $(-6)^2 - 4(9)(1)$
$b = -6$ $36 - 36 = 0$
$c = 1$

one real solution

3. $x^2 + x + 1 = 0$

$a = 1$
$b = 1$ $1^2 - 4(1)(1)$
$c = 1$ -3

no real solutions

4. $x^2 + 3 = 2x$

$x^2 + 3 - 2x = 0$
$x^2 - 2x + 3 = 0$

$a = 1$ $(-2)^2 - 4(1)(3)$
$b = 2$ $4 - 12 = -8$
$c = 3$

no real solutions

The Quadratic Formula:

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve. Round to the nearest tenth, if necessary.

A. $2x^2 + 3x - 5 = 0$

$a = 2$
$b = 3$
$c = -5$

$X = \frac{-3 \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$

$X = \frac{-3 \pm \sqrt{49}}{4}$

$X = \frac{-3 \pm 7}{4}$

$X = \frac{-3+7}{4} = 1$ $X = \frac{-3-7}{4} = -2.5$

$\boxed{X = 1, -2.5}$

B. $2x = x^2 - 3$ $X^2 - 2x - 3 = 0$

$a = 1$
$b = -2$
$c = -3$

$X = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)}$

$X = \frac{2 \pm \sqrt{16}}{2}$

$X = \frac{2 \pm 4}{2}$

$X = \frac{2+4}{2} = 3$ $X = \frac{2-4}{2} = -1$

$\boxed{X = 3, -1}$

C. $x^2 - 2x - 4 = 0$

$a = 1$
$b = -2$
$c = -4$

$X = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-4)}}{2(1)}$

$X = \frac{2 \pm \sqrt{20}}{2}$

$X = \frac{2 \pm \sqrt{20}}{2} = 3.2$ $X = \frac{2 - \sqrt{20}}{2} = -1.2$

$\boxed{X = 3.2, -1.2}$

D. $2x^2 - 8x + 1 = 0$

$a = 2$
$b = -8$
$c = 1$

$X = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(1)}}{2(2)}$

$X = \frac{8 \pm \sqrt{56}}{4}$

$X = \frac{8 + \sqrt{56}}{4} = 15.0$ $X = \frac{8 - \sqrt{56}}{4} = .1$

$\boxed{X = 15.0, .1}$

Exact answer:
$X = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$

Exact answer:
$X = \frac{8 \pm 2\sqrt{14}}{4} = \frac{4 \pm \sqrt{14}}{2} = 2 \pm \frac{1}{2}\sqrt{14}$

Your turn.

Find the number of real solutions. $b^2 - 4ac$

A. $x^2 + 7x - 5 = 0$

$a = 1$
$b = 7$
$c = -5$

$7^2 - 4(1)(-5) = 69$

two real solutions

B. $x^2 - 15 = 0$

$a = 1$
$b = 0$
$c = -15$

$0^2 - 4(1)(-15) = 60$

two real solutions

C. $2x^2 + 16 = 5x$

$2x^2 - 5x + 16 = 0$

$a = 2$
$b = -5$
$c = 16$

$(-5)^2 - 4(2)(16) = -103$

no real solutions

D. $9x^2 + 12x = -4$

$9x^2 + 12x + 4 = 0$

$a = 9$
$b = 12$
$c = 4$

$12^2 - 4(9)(4) = 0$

one real solution

Solve. Round to the nearest tenth, if necessary. (easy) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a. $x^2 - 5x + 4 = 0$

$a = 1$
$b = -5$
$c = 4$

$x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(1)(4)}}{2(1)}$

$x = \dfrac{5 \pm \sqrt{9}}{2}$

$x = \dfrac{5 \pm 3}{2}$

$x = \dfrac{5+3}{2} = 4 \qquad x = \dfrac{5-3}{2} = 1$

$x = 4, 1$

Note: Factoring is better choice
$(x-4)(x-1) = 0$
$x = 4, 1$

b. $x^2 + 8x + 15 = 0$

$a = 1$
$b = 8$
$c = 15$

$x = \dfrac{-8 \pm \sqrt{8^2 - 4(1)(15)}}{2(1)}$

$x = \dfrac{-8 \pm \sqrt{4}}{2}$

$x = \dfrac{-8 \pm 2}{2}$

$x = \dfrac{-8+2}{2} = -3 \qquad x = \dfrac{-8-2}{2} = -5$

$x = -3, -5$

Note: Factoring is better choice
$(x+5)(x+3) = 0$
$x = -5, -3$

c. $x^2 + x + 3 = 0$

$a = 1$
$b = 1$
$c = 3$

$X = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2(1)}$

$X = \dfrac{-1 \pm \sqrt{-11}}{2}$

$\boxed{\text{no real solution}}$

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d. $x^2 = -14x - 40$

$x^2 + 14x + 40 = 0$

*Better to factor
$(x+10)(x+4) = 0$
$x = -10, -4$

$a = 1$
$b = 14$
$c = 40$

$X = \dfrac{-14 \pm \sqrt{14^2 - 4(1)(40)}}{2(1)}$

$X = \dfrac{-14 \pm \sqrt{36}}{2}$

$X = \dfrac{-14 \pm 6}{2}$ $\boxed{X = -4, -10}$

$X = \dfrac{-14 + 6}{2} = \cancel{} -4$ $X = \dfrac{-14 - 6}{2} = -10$

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e. $3x^2 + 14x - 5 = 0$ *could factor

$a = 3$
$b = 14$
$c = -5$

$X = \dfrac{-14 \pm \sqrt{14^2 - 4(3)(-5)}}{2(3)}$

$X = \dfrac{-14 \pm \sqrt{256}}{6}$

$X = \dfrac{-14 \pm 16}{6}$ $\boxed{X = \frac{1}{3}, -5}$

$X = \dfrac{-14 + 16}{6} = \dfrac{1}{3}$ $X = \dfrac{-14 - 16}{6} = -5$

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f. $x^2 - 6x - 7 = 0$

$a = 1$
$b = -6$
$c = -7$

$X = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(-7)}}{2(1)}$

$X = \dfrac{6 \pm \sqrt{64}}{2}$

$X = \dfrac{6 \pm 8}{2}$ $\boxed{X = 7, -1}$

$X = 6 + 8 = 7$ $X = \dfrac{6-8}{2} = -1$

*Better to factor
$(x-7)(x+1) = 0$
$X = 7, -1$

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g. $2x^2 = 7x - 3$ *could factor

$0 = 2x^2 - 7x + 3$

$a = 2$
$b = -7$
$c = 3$

$X = \dfrac{7 \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$

$X = \dfrac{7 \pm \sqrt{25}}{4}$

$X = \dfrac{7 \pm 5}{4}$

$X = \dfrac{7 + 5}{4} = 3$ $X = \dfrac{7 - 5}{4} = \dfrac{1}{2}$

$\boxed{X = 3, \frac{1}{2}}$

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h. $6x^2 + x - 1 = 0$ *could factor

$a = 6$
$b = 1$
$c = -1$

$X = \dfrac{-1 \pm \sqrt{1^2 - 4(6)(-1)}}{2(6)}$

$X = \dfrac{-1 \pm \sqrt{25}}{12}$

$X = \dfrac{-1 \pm 5}{12}$

$X = \dfrac{-1 + 5}{12} = \dfrac{1}{3}$ $X = \dfrac{-1 - 5}{12} = -\dfrac{1}{2}$

$\boxed{X = \frac{1}{3}, -\frac{1}{2}}$