25.	$\ln(4x-7)=\ln(x+11)$ $4x-7=x+11$ $3x=18$ $x=6$ Check: $\ln(4x-7)=\ln(x+11)$ $\ln(4\cdot 6-7)\stackrel{?}{=}\ln(6+11)$ $\ln 17=\ln 17$ ✓
26.	$\ln(x+19)=\ln(7x-8)$ $x+19=7x-8$ $27=6x$ $\frac{9}{2}=x$ Check: $\ln(x+19)=\ln(7x-8)$ $\ln\left(\frac{9}{2}+19\right)\stackrel{?}{=}\ln\left(7\cdot\frac{9}{2}-8\right)$ $\ln\frac{47}{2}=\ln\frac{47}{2}$ ✓
27.	$\log_5(2x-7)=\log_5(3x-9)$ $2x-7=3x-9$ $2=x$ Check: $\log_5(2x-7)=\log_5(3x-9)$ $\log_5(2\cdot 2-7)\stackrel{?}{=}\log_5(3\cdot 2-9)$ $\log_5(-3)\stackrel{?}{=}\log_5(-3)$ Because $\log_5(-3)$ is *not* defined, 2 is not a solution. So, there is no solution.

28.	$\log(12x - 11) = \log(3x + 13)$ $12x - 11 = 3x + 13$ $9x = 24$ $x = \frac{8}{3}$ Check: $\log(12x - 11) = \log(3x + 13)$ $\log\left(12 \cdot \frac{8}{3} - 11\right) \stackrel{?}{=} \log\left(3 \cdot \frac{8}{3} + 13\right)$ $\log 21 = \log 21$ ✓ The solution is $\frac{8}{3}$.
29.	$\log_3(18x + 7) = \log_3(3x + 38)$ $18x + 7 = 3x + 38$ $15x = 31$ $x = \frac{31}{15}$ Check: $\log_3(18x + 7) = \log_3(3x + 38)$ $\log_3\left(18 \cdot \frac{31}{15} + 7\right) \stackrel{?}{=} \log_3\left(3 \cdot \frac{31}{15} + 38\right)$ $\log_3 \frac{221}{5} = \log_3 \frac{221}{5}$ ✓ The solution is $\frac{31}{15}$.
30.	$\log_6(3x - 10) = \log_6(14 - 5x)$ $3x - 10 = 14 - 5x$ $8x = 24$ $x = 3$ Check: $\log_6(3x - 10) = \log_6(14 - 5x)$ $\log_6(3 \cdot 3 - 10) \stackrel{?}{=} \log_6(14 - 5 \cdot 3)$ $\log_6(-1) \stackrel{?}{=} \log_6(-1)$ Because $\log_6(-1)$ is not defined, 3 is *not* a solution. So, there is no solution.

<table>
<tr><td>31.</td><td>$\log_8(5-12x)=\log_8(6x-1)$
$5-12x=6x-1$
$6=18x$
$\frac{1}{3}=x$
Check: $\log_8(5-12x)=\log_8(6x-1)$
$\log_8\left(5-12\cdot\frac{1}{3}\right)\stackrel{?}{=}\log 8\left(6\cdot\frac{1}{3}-1\right)$
$\log_8 1=\log_8 1$ ✓</td></tr>
<tr><td>32.</td><td>$\log_4 x=-1$
$4^{\log_4 x}=4^{-1}$
$x=\frac{1}{4}$
Check: $\log_4 x=\log_4\frac{1}{4}$
Because $4^{-1}=\frac{1}{4}$, $\log_4\frac{1}{4}=-1$ ✓</td></tr>
<tr><td>33.</td><td>$5\ln x=35$
$\ln x=7$
$e^{\ln x}=e^7$
$x=e^7\approx 1096.63$
Check: $5\ln x=5\ln e^7=5\cdot 7=35$ ✓</td></tr>
</table>

34.	$\frac{1}{3}\log_5 12x = 2$ $\log_5 12x = 6$ $5^{\log_5 12x} = 5^6$ $12x = 15{,}625$ $x = \frac{15{,}625}{12}$ Check: $\frac{1}{3}\log_5 12x = 2$ $\frac{1}{3}\log_5 12\left(\frac{15{,}625}{12}\right) \stackrel{?}{=} 2$ $\frac{1}{3}\log_5 15{,}625 \stackrel{?}{=} 2$ $\log_5 (15{,}625)^{1/3} \stackrel{?}{=} 2$ $\log_5 25 = 2$ ✓
35.	$5.2\log_4 2x = 16$ $\log_4 2x \approx 3.0769$ $4^{\log_4 2x} \approx 4^{3.0769}$ $2x \approx 71.20$ $x \approx 35.60$ Check: $5.2\log_4 2x = 16$ $5.2\log_4 2 \cdot 35.60 \stackrel{?}{=} 16$ $5.2\log_4 71.2 \stackrel{?}{=} 16$ $5.2\left(\frac{\log 71.2}{\log 4}\right) \stackrel{?}{=} 16$ $5.2(3.0769) \stackrel{?}{=} 16$ $16.0 = 16$ ✓

36.	$\log_2(x-4) = 6$ $2^{\log_2(x-4)} = 2^6$ $x - 4 = 64$ $x = 68$ Check: $\log_2(x-4) = \log_2(68-4) = \log_2 64$ Because $2^6 = 64$, $\log_2 64 = 6$.
37.	$\log_2 x + \log_2(x-2) = 3$ $\log_2[x(x-2)] = 3$ $2^{\log_2[x(x-2)]} = 2^3$ $x(x-2) = 8$ $x^2 - 2x - 8 = 0$ $(x-4)(x+2) = 0$ $x = 4 \quad \text{or} \quad x = -2$ Check: $\log_2 x + \log_2(x-2) = 3$ $\log_2 4 + \log_2(4-2) \stackrel{?}{=} 3$ $\log_2 4 + \log_2 2 \stackrel{?}{=} 3$ $\log_2 8 \stackrel{?}{=} 3$ $\log_2 2^3 \stackrel{?}{=} 3$ $3 = 3$ ✓ $\log_2 x + \log_2(x-2) = 3$ $\log_2(-2) + \log_2(-2-2) \stackrel{?}{=} 3$ $\log_2(-2) + \log_2(-4) \stackrel{?}{=} 3$ Because $\log_2(-2)$ and $\log_2(-4)$ are *not* defined, -2 is not a solution.

38.	$\log_4(-x) + \log_4(x+10) = 2$ $\log_4[-x(x+10)] = 2$ $4^{\log 4\,[-x(x+10)]} = 4^2$ $-x(x+10) = 16$ $-x^2 - 10x = 16$ $0 = x^2 + 10x + 16$ $0 = (x+8)(x+2)$ $x = -8 \quad \text{or} \quad x = -2$ Check: $\log_4(-x) + \log_4(x+10) = 2$ $\log_4(-(-8)) + \log_4(-8+10) \stackrel{?}{=} 2$ $\log_4 8 + \log_4 2 \stackrel{?}{=} 2$ $\log_4 16 \stackrel{?}{=} 2$ $\log_4 4^2 \stackrel{?}{=} 2$ $2 = 2$ ✓ $\log_4(-x) + \log_4(x+10) = 2$ $\log_4(-(-2)) + \log_4(-2+10) \stackrel{?}{=} 2$ $\log_4 2 + \log_4 8 \stackrel{?}{=} 2$ $\log_4 16 \stackrel{?}{=} 2$ $\log_4 4^2 \stackrel{?}{=} 2$ $2 = 2$ ✓ The solutions are -8 and -2.

39.

$$\ln(x+3) + \ln x = 1$$
$$\ln[x(x+3)] = 1$$
$$e^{\ln[x(x+3)]} = e^1$$
$$x(x+3) = e$$
$$x^2 + 3x = e$$
$$x^2 + 3x - e = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{9 - 4(-e)}}{2} = \frac{-3 \pm \sqrt{9 + 4e}}{2}$$
$$x = \frac{-3 - \sqrt{9 + 4e}}{2} \quad \text{or} \quad x = \frac{-3 + \sqrt{9 + 4e}}{2}$$

Check: $\ln(x+3) + \ln x = 1$

$$\ln\left(\frac{-3 - \sqrt{9+4e}}{2} + 3\right) + \ln\left(\frac{-3 - \sqrt{9+4e}}{2}\right) \stackrel{?}{=} 1$$
$$\ln\left(\frac{3 - \sqrt{9+4e}}{2}\right) + \ln\left(\frac{-3 - \sqrt{9+4e}}{2}\right) \stackrel{?}{=} 1$$
$$\ln(-0.73) + \ln(-3.73) \stackrel{?}{=} 1$$

Because $\ln(-0.73)$ and $\ln(-3.73)$ are not defined, $\frac{3 - \sqrt{9+4e}}{2}$ is *not* a solution.

$$\ln(x+3) + \ln x = 1$$
$$\ln\left(\frac{-3 + \sqrt{9+4e}}{2} + 3\right) + \ln\left(\frac{-3 + \sqrt{9+4e}}{2}\right) \stackrel{?}{=} 1$$
$$\ln\left(\frac{3 + \sqrt{9+4e}}{2}\right) + \ln\left(\frac{-3 + \sqrt{9+4e}}{2}\right) \stackrel{?}{=} 1$$
$$\ln\left(\frac{-9 + 9 + 4e}{4}\right) \stackrel{?}{=} 1$$
$$\ln(e) \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

So, the solution is $\frac{-3 + \sqrt{9+4e}}{2}$.

40.	$4\ln(-x) + 3 = 21$ $4\ln(-x) = 18$ $\ln(-x) = \frac{9}{2}$ $e^{\ln(-x)} = e^{9/2}$ $-x \approx e^{9/2}$ $x = -e^{9/2} \approx -90.02$ Check: $4\ln(-x) + 3 \stackrel{?}{=} 21$ $4\ln\left(-\left(-e^{9/2}\right)\right) + 3 \stackrel{?}{=} 21$ $4\ln\left(e^{9/2}\right) + 3 \stackrel{?}{=} 21$ $4\left(\frac{9}{2}\right) + 3 \stackrel{?}{=} 21$ $21 = 21$ ✓

41.

$$\log_5(x+4) + \log_5(x+1) = 2$$

$$\log_5[(x+4)(x+1)] = 2$$

$$5^{\log_5[(x+4)(x+1)]} = 5^2$$

$$(x+4)(x+1) = 25$$

$$x^2 + 5x + 4 = 25$$

$$x^2 + 5x - 21 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4(-21)}}{2} = \frac{-5 \pm \sqrt{109}}{2}$$

$$x = \frac{-5 - \sqrt{109}}{2} \quad \text{or} \quad x = \frac{-5 + \sqrt{109}}{2}$$

Check: $\log_5(x+4) + \log_5(x+1) = 2$

$$\log_5\left(\frac{-5 - \sqrt{109}}{2} + 4\right) + \log_5\left(\frac{-5 - \sqrt{109}}{2} + 1\right) \stackrel{?}{=} 2$$

$$\log_5\left(\frac{3 - \sqrt{109}}{2}\right) + \log_5\left(\frac{-3 - \sqrt{109}}{2}\right) \stackrel{?}{=} 2$$

$$\log_5(-7.44) + \log_5(-6.72) \stackrel{?}{=} 2$$

Because $\log_5(-7.44)$ and $\log_5(-6.72)$ are not defined, $\frac{-5 - \sqrt{109}}{2}$ is not a solution.

$$\log_5(x+4) + \log_5(x+1) = 2$$

$$\log_5\left(\frac{-5 + \sqrt{109}}{2} + 4\right) + \log_5\left(\frac{-5 + \sqrt{109}}{2} + 1\right) \stackrel{?}{=} 2$$

$$\log_5\left(\frac{3 + \sqrt{109}}{2}\right) + \log_5\left(\frac{-3 + \sqrt{109}}{2}\right) \stackrel{?}{=} 2$$

$$\log_5\left(\frac{109 - 9}{4}\right) \stackrel{?}{=} 2$$

$$\log_5 25 \stackrel{?}{=} 2$$

$$2 = 2 \checkmark$$

So, $\frac{-5 + \sqrt{109}}{2}$ is a solution.

42.	$\log_6 3x + \log_6 (x-1) = 3$ $\log_6 [3x(x-1)] = 3$ $6^{\log_6 [3x(x-1)]} = 6^3$ $3x(x-1) = 216$ $3x^2 - 3x = 216$ $3x^2 - 3x - 216 = 0$ $x^2 - x - 72 = 0$ $(x-9)(x+8) = 0$ $x = 9 \quad \text{or} \quad x = -8$ Check: $\log_6 3x + \log_6 (x-1) = 3$ $\log_6 (3 \cdot 9) + \log_6 (9-1) \stackrel{?}{=} 3$ $\log_6 27 + \log_6 8 \stackrel{?}{=} 3$ $\log_6 216 \stackrel{?}{=} 3$ $\log_6 6^3 \stackrel{?}{=} 3$ $3 = 3$ ✓ $\log_6 3x + \log_6 (x-1) = 3$ $\log_6(3(-8)) + \log_6(-8-1) \stackrel{?}{=} 3$ $\log_6(-24) + \log_6(-9) \stackrel{?}{=} 3$ Because $\log_6(-24)$ and $\log_6(-9)$ are not defined, -8 is not a solution. The solution is 9.

43.

$$\log_3(x-9)+\log_3(x-3)=2$$

$$\log_3[(x-9)(x-3)]=2$$

$$3^{\log_3[(x-9)(x-3)]}=3^2$$

$$(x-9)(x-3)=9$$

$$x^2-12x+27=9$$

$$x^2-12x+18=0$$

$$x=\frac{12\pm\sqrt{144-4(18)}}{2}=\frac{12\pm\sqrt{72}}{2}=6\pm3\sqrt{2}$$

$x=6-3\sqrt{2}$ or $x=6+3\sqrt{2}$

Check: $\log_3(x-9)+\log_3(x-3)=2$

$$\log_3(6-3\sqrt{2}-9)+\log_3(6-3\sqrt{2}-3)\stackrel{?}{=}2$$

$$\log_3(-3-3\sqrt{2})+\log_3(3-3\sqrt{2})\stackrel{?}{=}2$$

$$\log_3(-7.25)+\log_3(-1.24)\stackrel{?}{=}2$$

Because $\log_3(-7.25)$ and $\log_3(-1.24)$ are not defined, $6-3\sqrt{2}$ is not a solution.

$$\log_3(x-9)+\log_3(x-3)=2$$

$$\log_3(6+3\sqrt{2}-9)+\log_3(6+3\sqrt{2}-3)\stackrel{?}{=}2$$

$$\log_3[(3\sqrt{2})^2-32]+\log_3(3+3\sqrt{2})\stackrel{?}{=}2$$

$$\log_3[(3\sqrt{2})^2-3^2]\stackrel{?}{=}2$$

$$\log_3 3^2\stackrel{?}{=}2$$

$$2=2\ \checkmark$$

So, the solution is $6+3\sqrt{2}$.